curacy there can be a reference to section 12.3.4, which, if the length of this volume is any indication, must be fully 3000 pages away.

MALCOLM C. HARRISON

Courant Institute of Mathematical Sciences New York University New York, New York 10012

19[3].—JOAN R. WESTLAKE, A Handbook of Numerical Matrix Inversion and Solution of Linear Equations, John Wiley & Sons, Inc., New York, 1968, viii + 171 pp., 23 cm. Price \$10.95.

This is, as the title indicates, a handbook. A two-page introduction is followed by brief descriptions of the standard direct and iterative methods for a total of 85 pages, with no theory and (at this stage) no evaluation. This is followed by chapters on measures of condition (four pages), measures of error (five pages), scaling (two pages), operational counts (four pages), comments and comparisons (14 pages), including some test results. In the appendix are a glossary (nine pages), a collection of basic theorems (ten pages), and a set of test matrices, and finally a list of references (126 items), a table of symbols, and an index. No programs are given. Each method is briefly but clearly described and the selection is quite reasonable. It should be a useful and convenient reference for the purpose intended.

A. S. H.

20[3].—ANDRÉ KORGANOFF & MONICA PAVEL-PARVU, Éléments de la Théorie des Matrices Carrées et Rectangles en Analyse Numérique, Dunod, Paris, 1967, xx + 441 pp., 25 cm. Price 98 F.

This is the second volume in a series entitled "Méthodes de calcul numérique," of which the first, *Algèbre non linéaire*, appeared in 1961 as a collection of papers on the subject, edited by the senior author of this volume. The first volume provides a fairly elementary but rigorous development of methods available at that time for solving nonlinear equations and systems of equations, the most sophisticated chapter being the first on error analysis.

The present volume, by contrast, treats only a limited aspect of the subject, but treats it in considerable depth and at a rather high level of sophistication. Primarily it is concerned with the Moore-Penrose generalized inverse, a subject which, along with still more general "generalized inverses," has recently led to a rapidly expanding literature. Presumably the eigenvalue problem will be the subject of a later volume.

The book is divided into three "parts." Of the three chapters in this part, the first provides a survey of certain notions from functional analysis, and the remaining two continue in a similar vein with the theory of norms as the guiding principle, these having been introduced already in the first chapter.

In Part 2, the first chapter starts out quite generally to discuss, for a given matrix a, the most general solutions of the equations  $e_g a = a$  and  $a e_d = a$ , thence the most general solutions of  $a'_g a = e_d$ , and  $a a'_d = e_g$ , and proceeds to impose

## BOOK REVIEWS

further conditions to arrive at the principal object of study, the Moore-Penrose generalized inverse. This is applied to the problem  $a \times b = c$  in the next chapter. The third chapter introduces a new approach (though already foreshadowed) in terms of minimizing norms of x and a x - c. However, the development is meager except for the Euclidean norm, which leads back again to Moore and Penrose. Finally, the last chapter describes a number of numerical methods for computing the generalized inverse.

There are no exercises, but there are numerical examples. A two-page general bibliography of items on matrices and functional analysis is given at the outset, and a rather extensive special bibliography is given at the end of each part. There is no index.

This is by no means an elementary text. But the numerical analyst with some degree of mathematical maturity can find here a great deal of interesting material. Although the literature on generalized inverses has expanded considerably since the book went to press, it gives a quite complete and systematic coverage of the theory up to that time, and the diverse points of approach suggest aspects of the subject that are by no means yet fully explored.

A. S. H.

21[3].—E. H. CUTHILL, Tables of Inverses and Determinants of Finite Segments of the Hilbert Matrix, Applied Mathematics Laboratory, Naval Ship Research & Development Center, Washington, D. C., ms. of 9 typewritten pp. + 346 computer sheets deposited in the UMT file.

The main table (Appendix A) gives on 326 computer sheets the exact (integer) values of the elements of the inverses of the first 37 segments of the Hilbert matrix. The symmetry of the inverse matrices is exploited through the printing of only those elements situated on or below the main diagonal. Included are the exact values of the determinants of these segments.

The underlying calculations were performed in variable-precision rational arithmetic on an IBM 360/50 system at the IBM Boston Programming Center, using programs written in PL/1-FORMAC. The program used in calculating the results in Appendix A is listed in Appendix B.

The exact values of the determinants of the segments of the Hilbert matrix were calculated independently from the well-known formula

$$\det H_n = \prod_{r=1}^{n-1} (r!)^4 / \prod_{r=1}^{2n-1} r! ,$$

corresponding to n = 2(1)62. (Beyond this point the range permitted by PL/1-FORMAC was exceeded.) These 61 numbers (which are reciprocals of integers) are separately tabulated in Appendix C, and the underlying program is listed in Appendix D.

Relevant mathematical formulas, as well as details of the computer calculations, are given in the introductory text, to which is appended a useful list of six references.

These extensive manuscript tables greatly exceed the range of similar earlier tables, such as those of Savage & Lukacs [1] and R. B. Smith [2], to which the present author refers.

J. W. W.